Twin Cities
ANSYS® User Meeting

November 2011

Mesh Discretization Error

… within Epsilon

Mesh Discretization Error

1. Mesh Discretization: The “One-sided” Error Source
2. Tet Pregidous
3. Case Study A: Shape-Functions Effect
    - With and without mid-side nodes
    - Stresses & Deflection
4. Case Study B: Mesh Convergence
    - Node vs. Element (Averaged vs Unaveraged)

    - PRERR (SEPC/SMXB)

Mesh Discretization Error

… within Epsilon

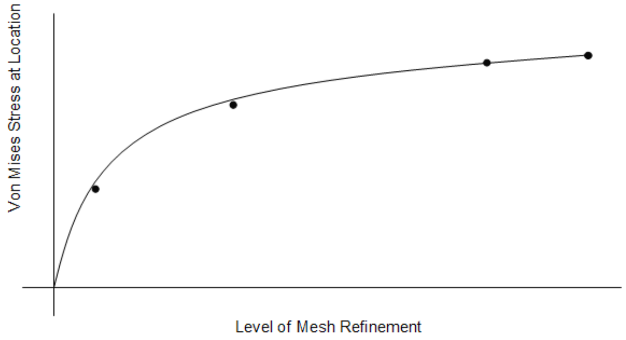
- Often small compared to load/material property error/scatter
- Ownership of error lands on analyst
  - Often linked to “credibility” of whole analysis
- True Error analysis would likely show Mesh Discretization is minor issue
  - And yet... Scrutiny continues
  - And rules and criteria abound… (while other scatter goes unmentioned)

“It can be *measured*? Well let’s fixate on it!”

Mesh Discretization Error

… within Epsilon

- A “one-sided” error source*
  - Predictions are usually ***lower*** than actual (not higher)
    - Excepting Singularities
  - Nagging feeling because of non-conservative nature
    - Stress is usually underpredicted*
  - Upper bound not determinable
    - Without employing knowledge of materials/loads/element shape functions – discussed later



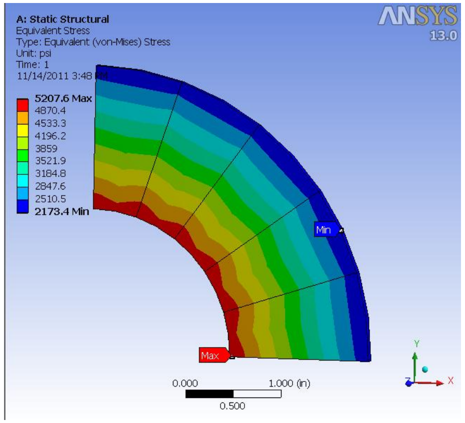
*Powergraphics results (classic) isn’t so one-sided – discussed later

Tet-Pregidious

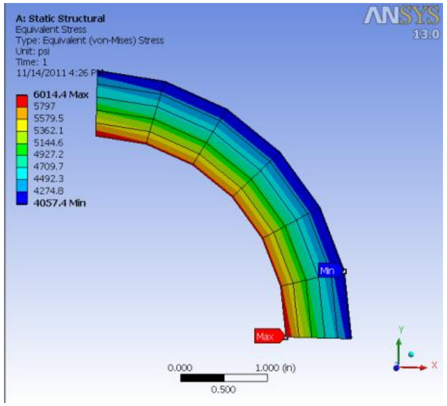
- Bias Against Tetrahedrons (Tet's)
  - Source of grievance?
    - Low order Tets (a.k.a "T4", a.k.a "non-midside noded Tet")
      - Too Stiff in bending / large error with 1 element through thickness
    - 1st Tet's (Berkely 1960's) were high order
    - You'd have to work at it to get ANSYS to create T4's (structural)
  - Tets (10 noded) are Less efficient per DOF
    - Longer solve times
    - Shorter meshing times
    - Added control allows refinement at location of interest
      - **More** efficient than Mapped meshing!
  - Less pleasing to the eye (esp. higher aspect ratios)
  - Stigmatism is receding over last decade

Case Study A

- Thick to thin rings with inner pressfit (radial expansion)
  - Stress gradient related to radius^2
- Case Study A, Expansion of Thick/Thin Ring
  - Actually used 5° wedge



A: Static Structural
Equivalent Stress
Type: Equivalent (von-Mises) Stress
Unit: psi
Time: 1
11/14/2011 3:48 PM



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Equivalent Stress
Type: Equivalent (von-Mises) Stress
Unit: psi
Time: 1
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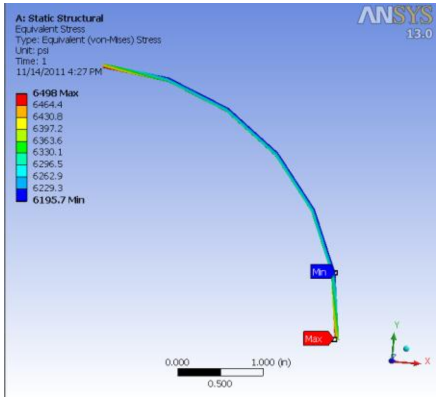
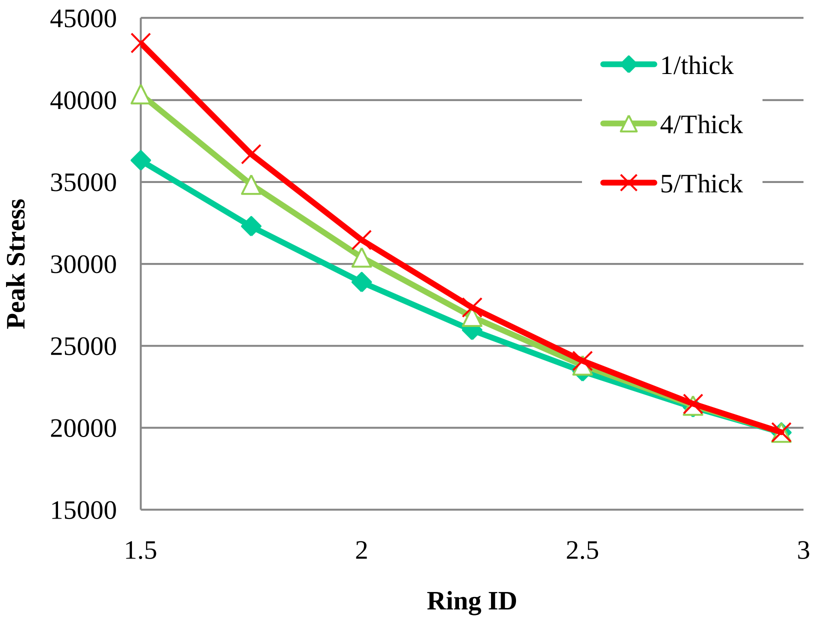


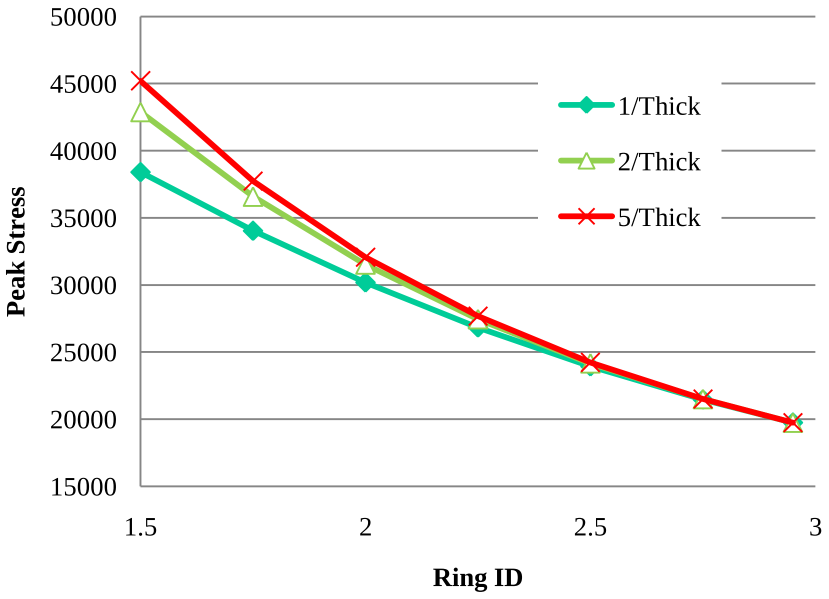
A: Static Structural
Equivalent Stress
Type: Equivalent (von-Mises) Stress
Unit: psi
Time: 1
11/14/2011 4:27 PM

Case Study A

- Case Study A
  - Peak Stresses have similar convergence patterns/rate



Low Order Elements

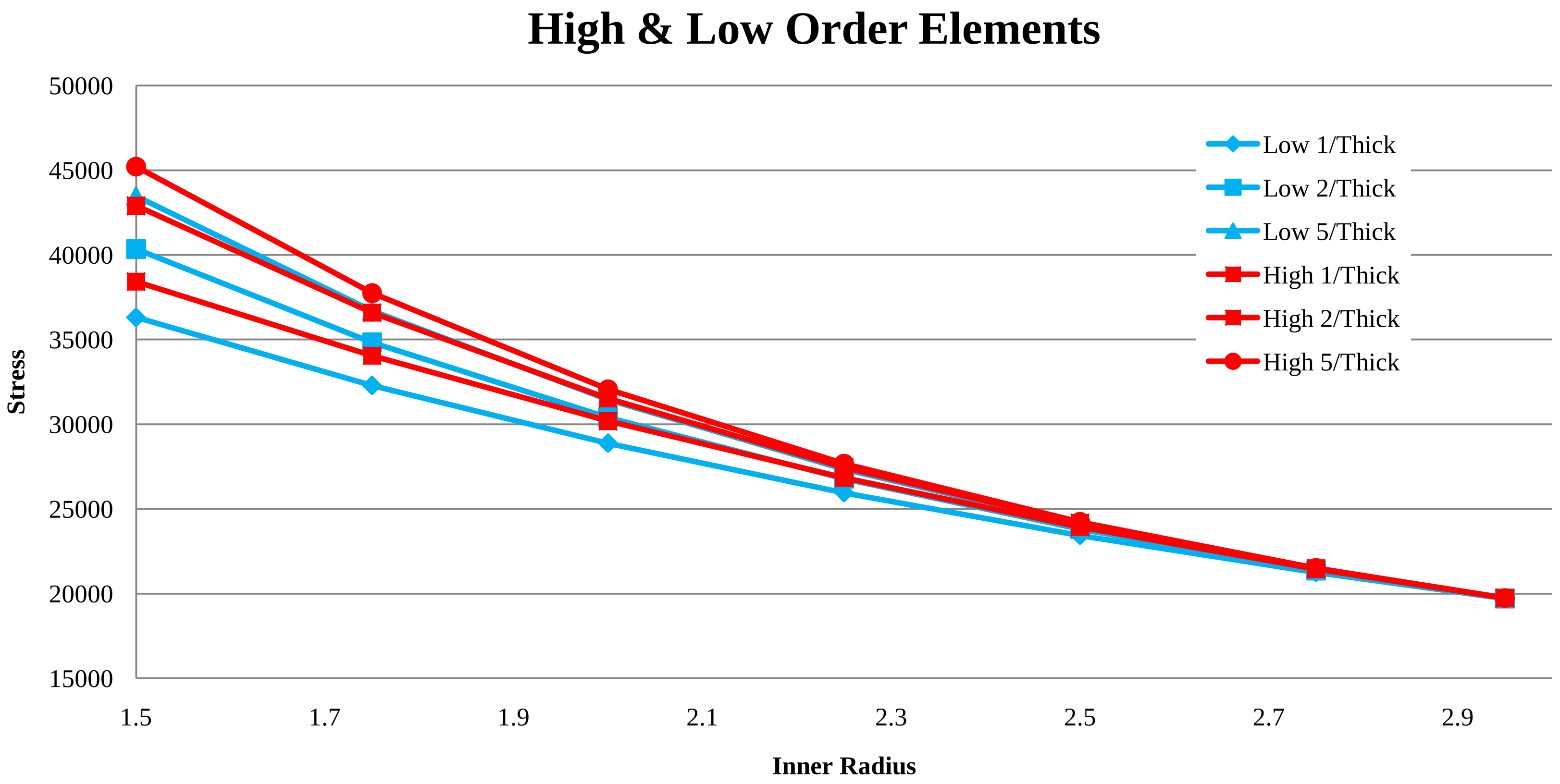


High Order Elements

Case Study A

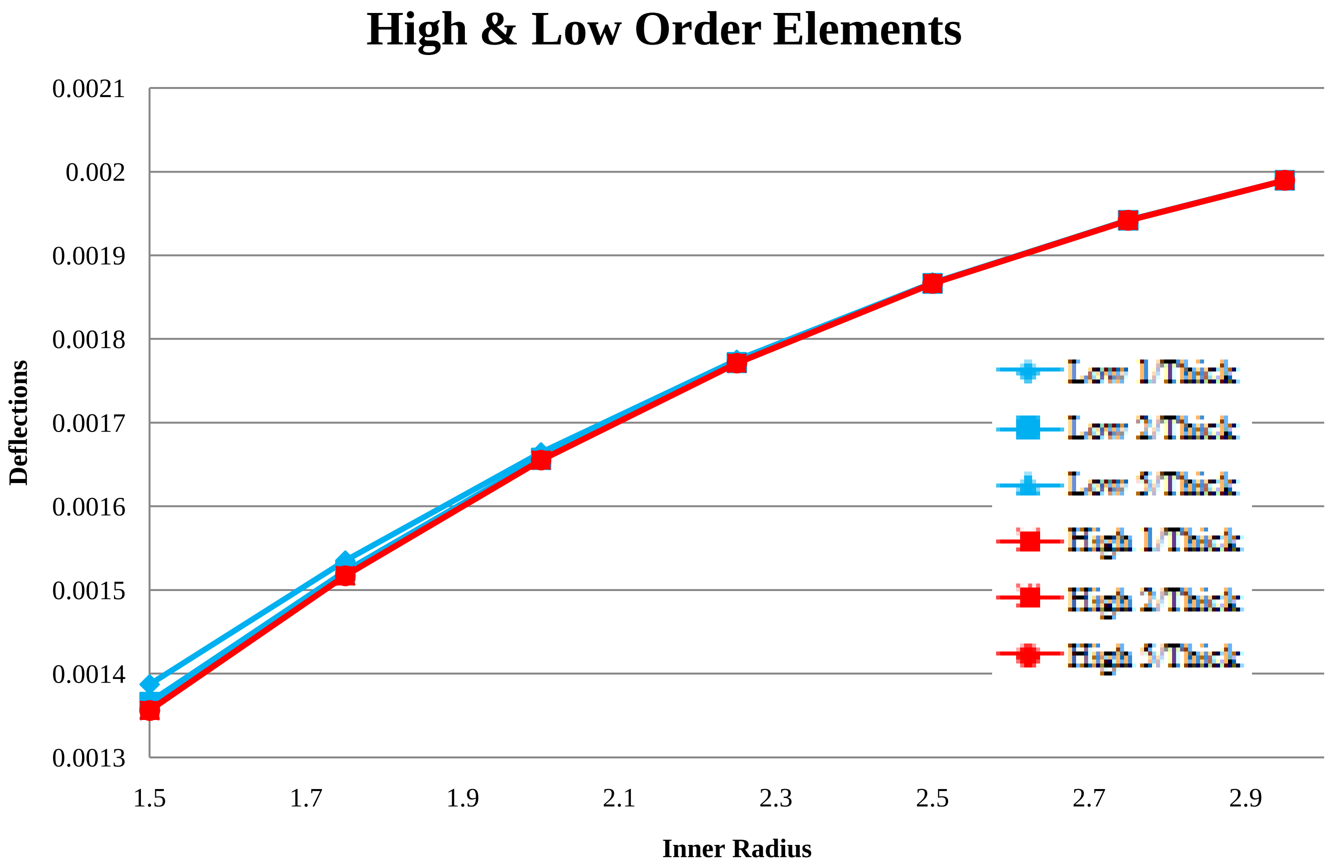
... within Epsilon

- Case Study A
  - Peak Stresses have similar convergence patterns/rate



Case Study A

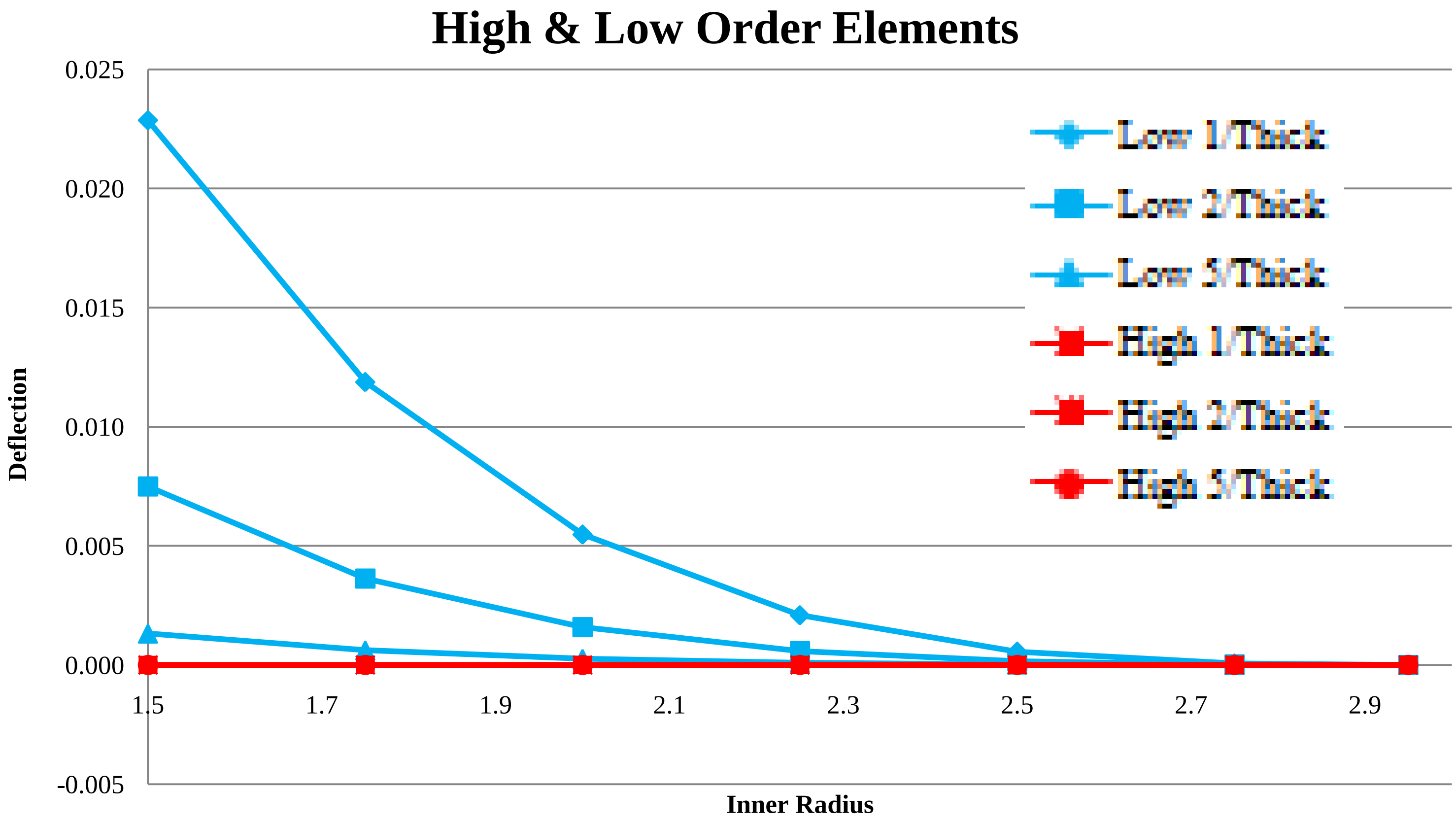
- Case Study A
  - OD Deflections



Case Study A

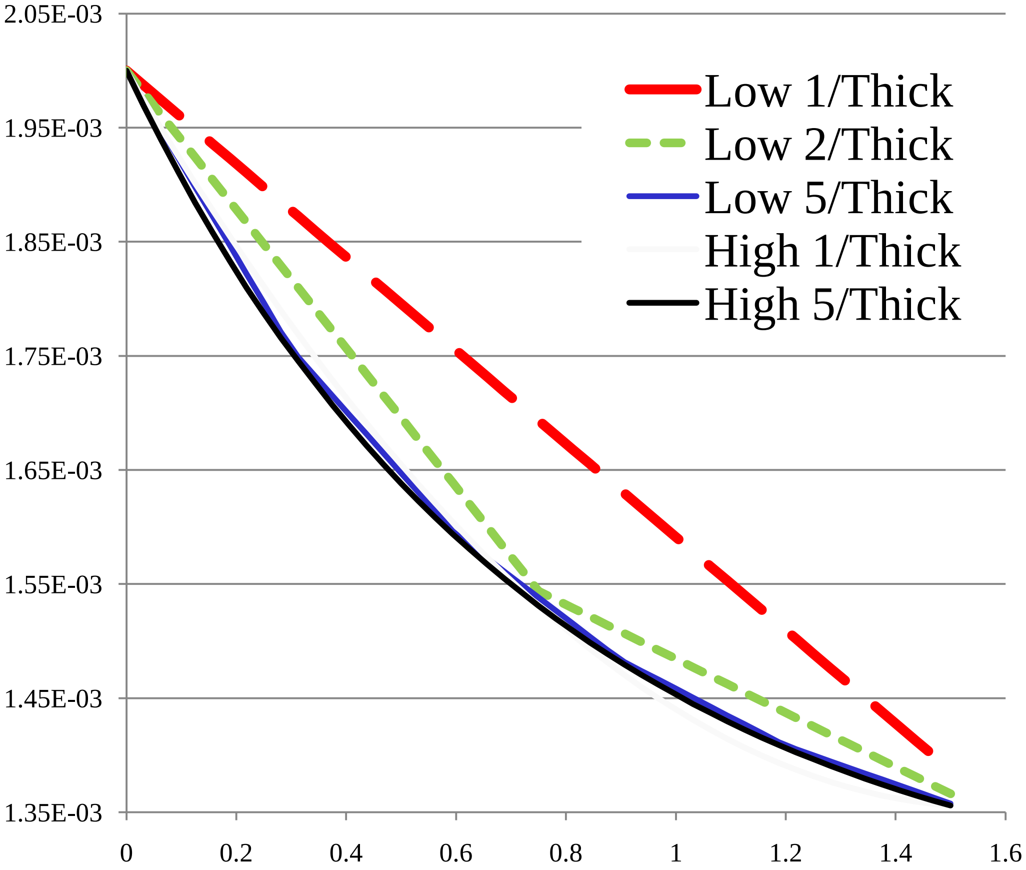
… within Epsilon

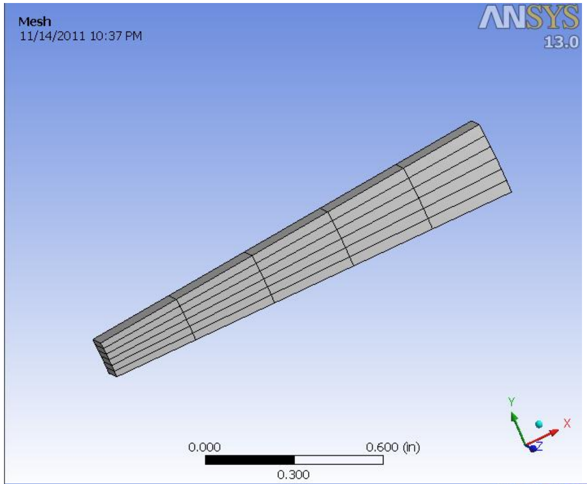
- Case Study A
  - OD Deflections



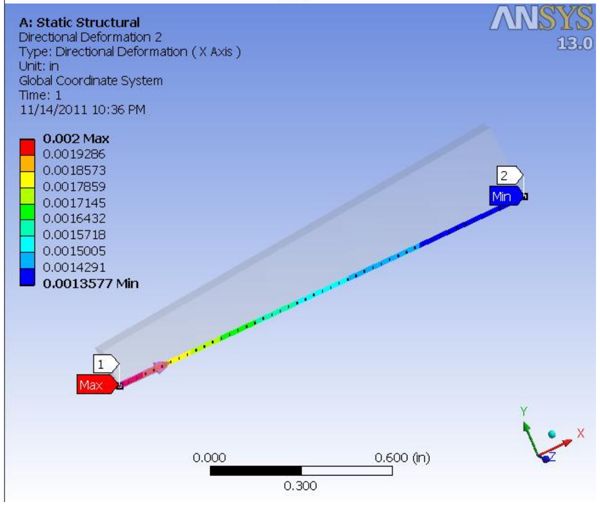
Case Study A

- Case Study A
  - Deflections Along Path





Mesh
11/14/2011 10:37 PM



A: Static Structural
Directional Deformation 2
Type: Directional Deformation (X Axis)
Unit: in
Global Coordinate System
Time: 1
11/14/2011 10:36 PM

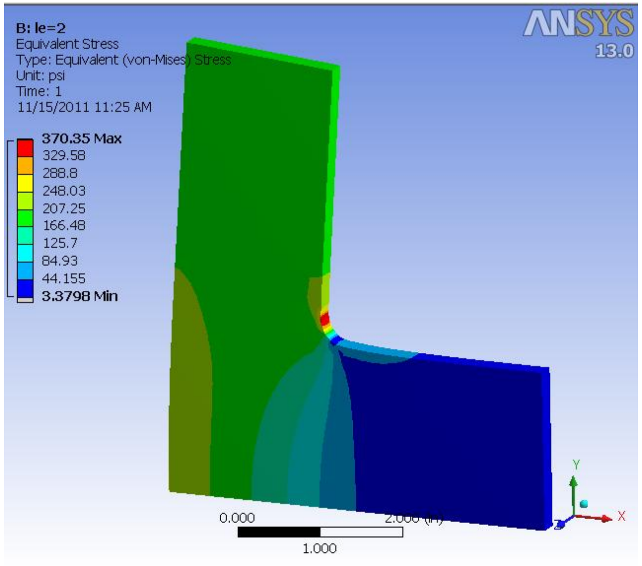
0.002 Max
0.0019286
0.0018573
0.0017859
0.0017145
0.0016432
0.0015718
0.0015005
0.0014291
0.0013577 Min

Case Study A

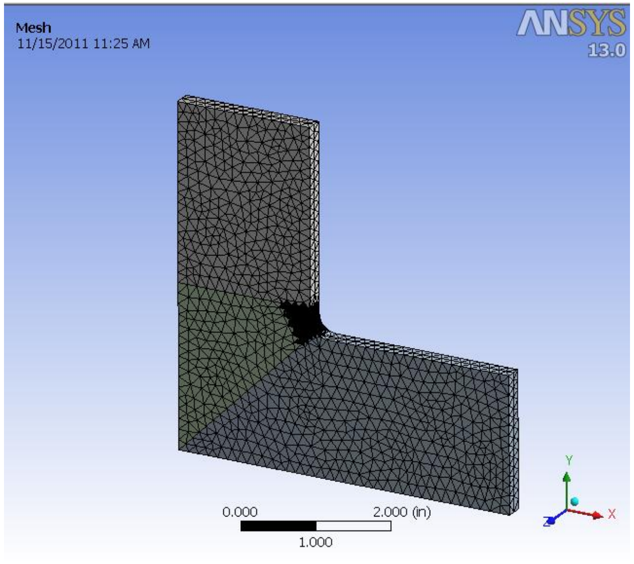
- Case Study A Conclusions
  - Element Stress Gradient
    - Linear for high or low order elements
  - Element Displacement Gradient
    - Linear for low order element
    - 2nd order polynomial for high order element
  - Thin Rings are well approximated with single element through the thickness
    - This extends to beams as well

Mesh Discretization Error

- Case Study B
  - Stress along path
  - Node vs. Element (Averaged vs Unaveraged)
  - PRERR (SEPC/SMXB)



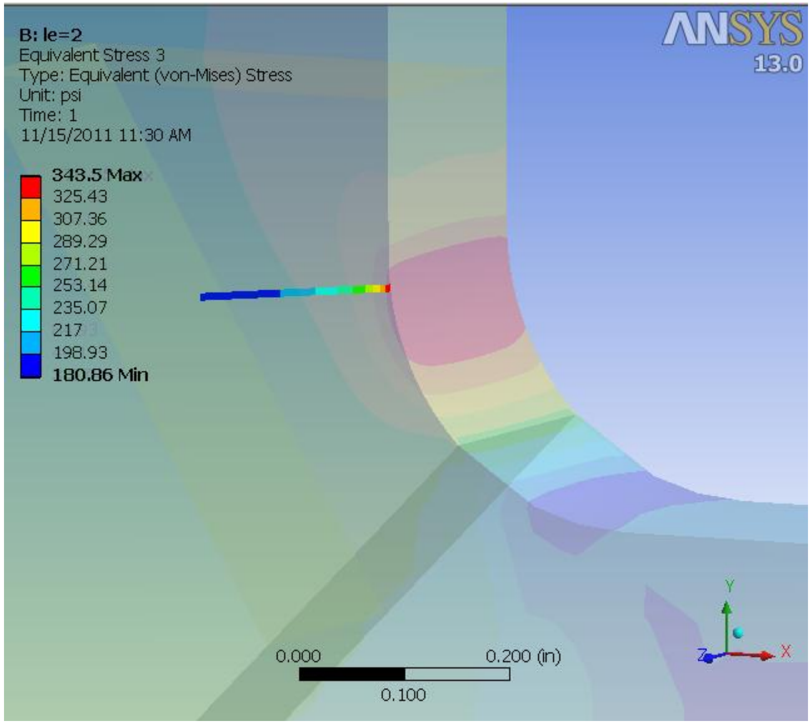
B: le=2
Equivalent Stress
Type: Equivalent (von-Mises) Stress
Unit: psi
Time: 1
11/15/2011 11:25 AM



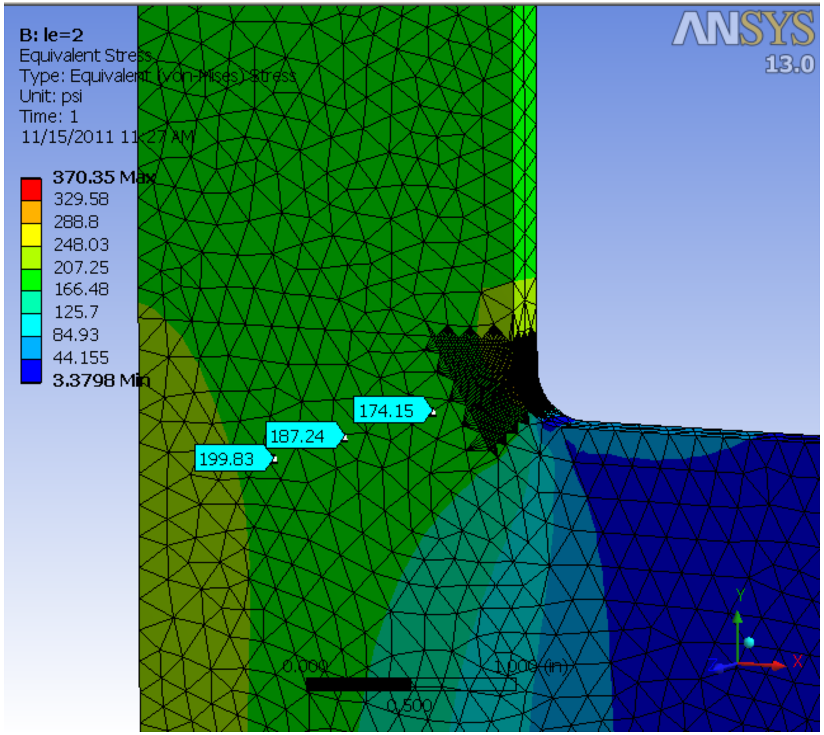
Mesh
11/15/2011 11:25 AM

Case Study B

- Stress along path
  - Background stress of 180
  - KT =2.0



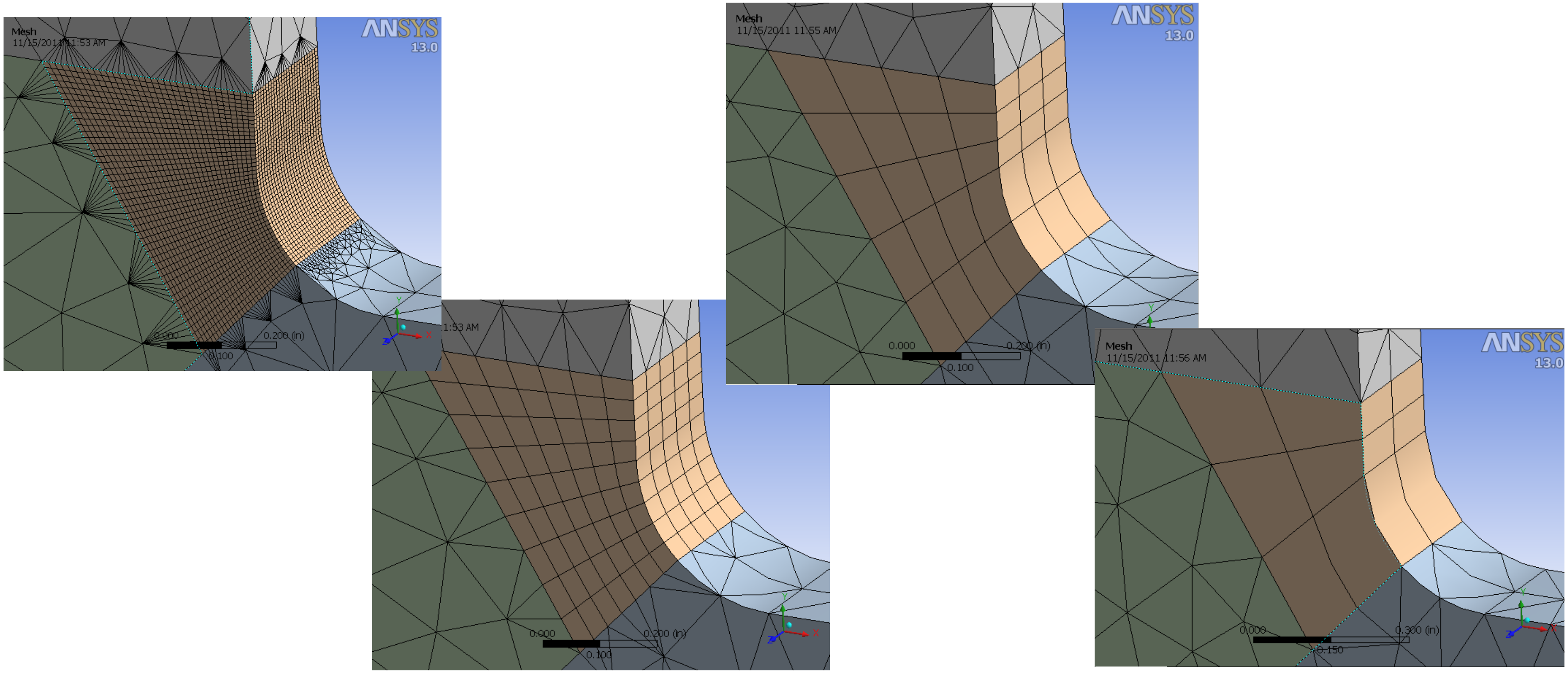
B: le=2
Equivalent Stress 3
Type: Equivalent (von-Mises) Stress
Unit: psi
Time: 1
11/15/2011 11:30 AM



B: le=2
Equivalent Stress
Type: Equivalent (von-Mises) Stress
Unit: psi
Time: 1
11/15/2011 11:27 AM

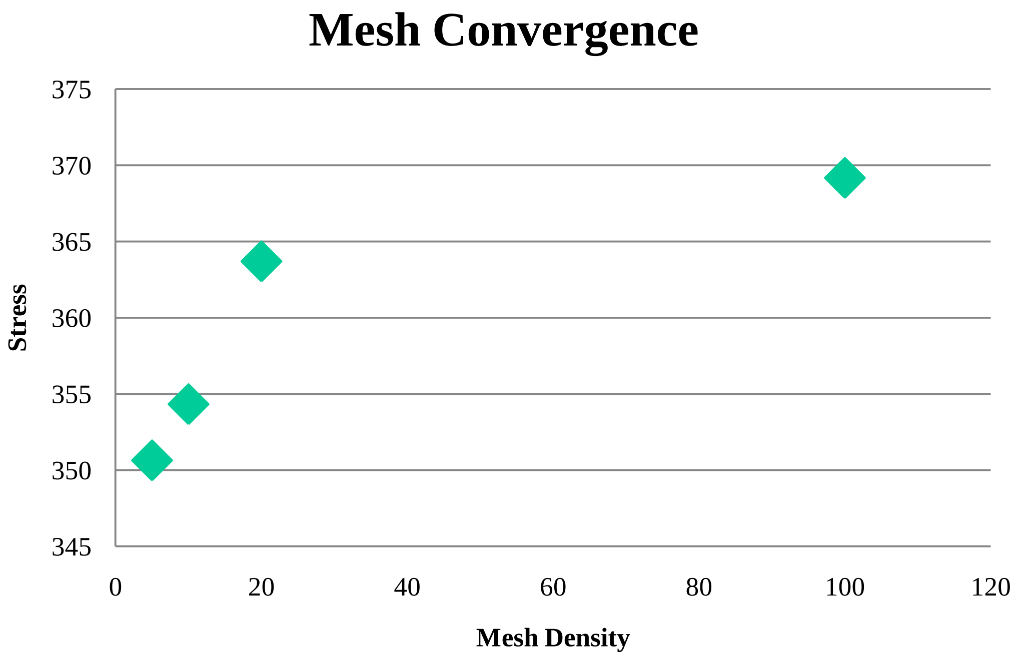
Case Study B

- Stress along path
  - Varying Mesh densities



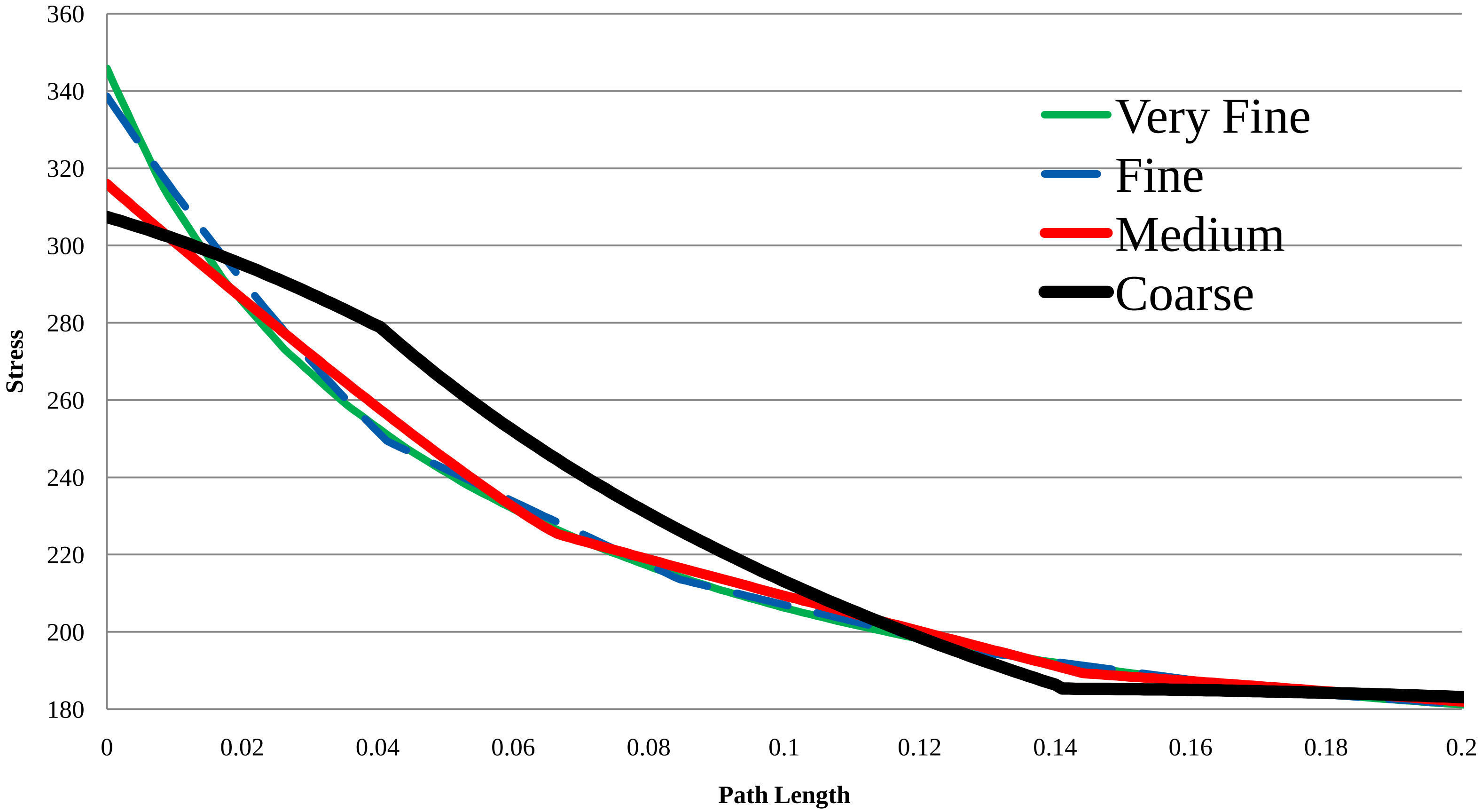
Case Study B

- Peak Stress
  - Varying Mesh densities
    - WB 's adaptive mesh refinement automates this task refining only regions of interest (thanks, paul)



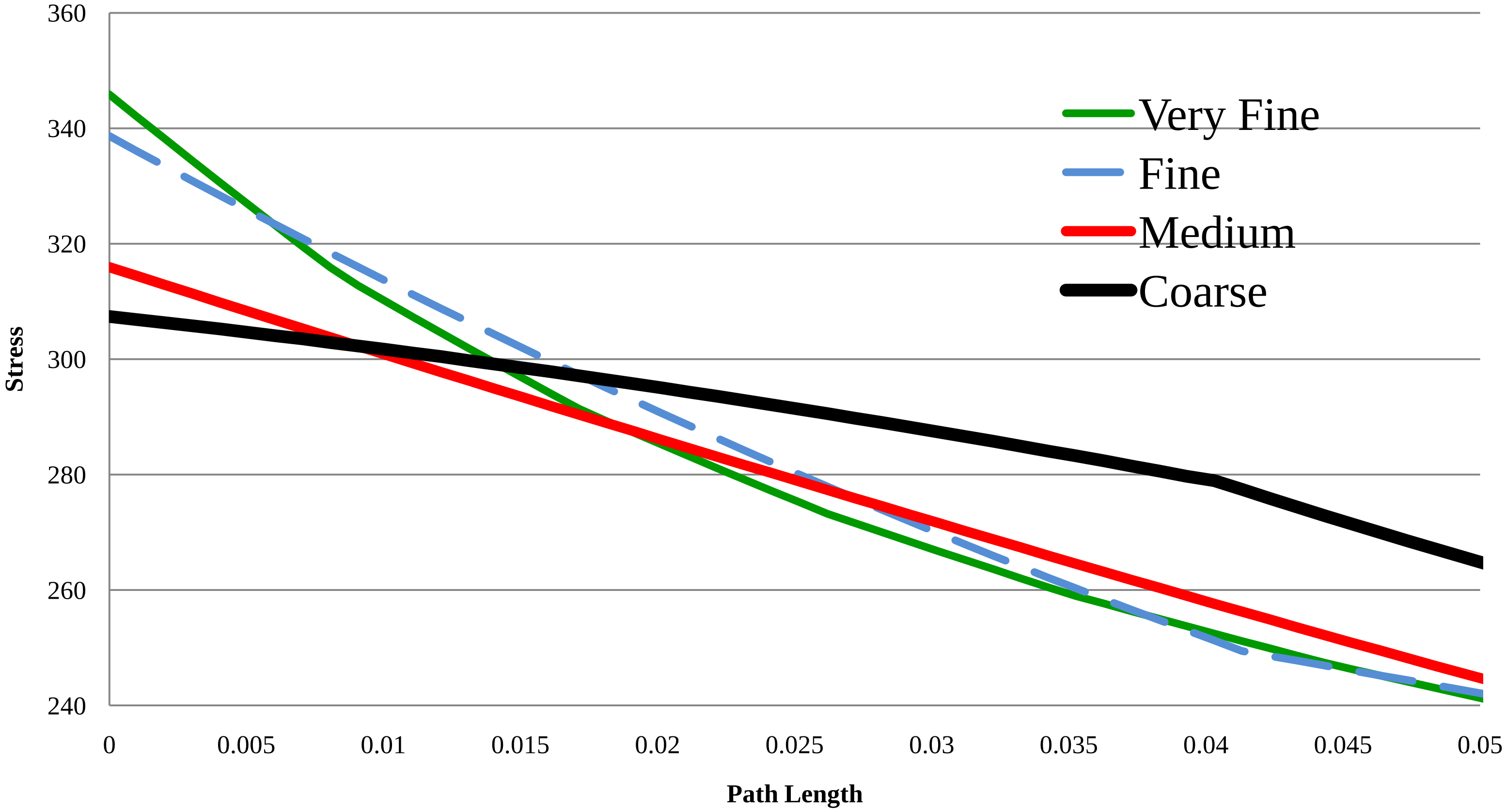
Case Study B

- Stress along path



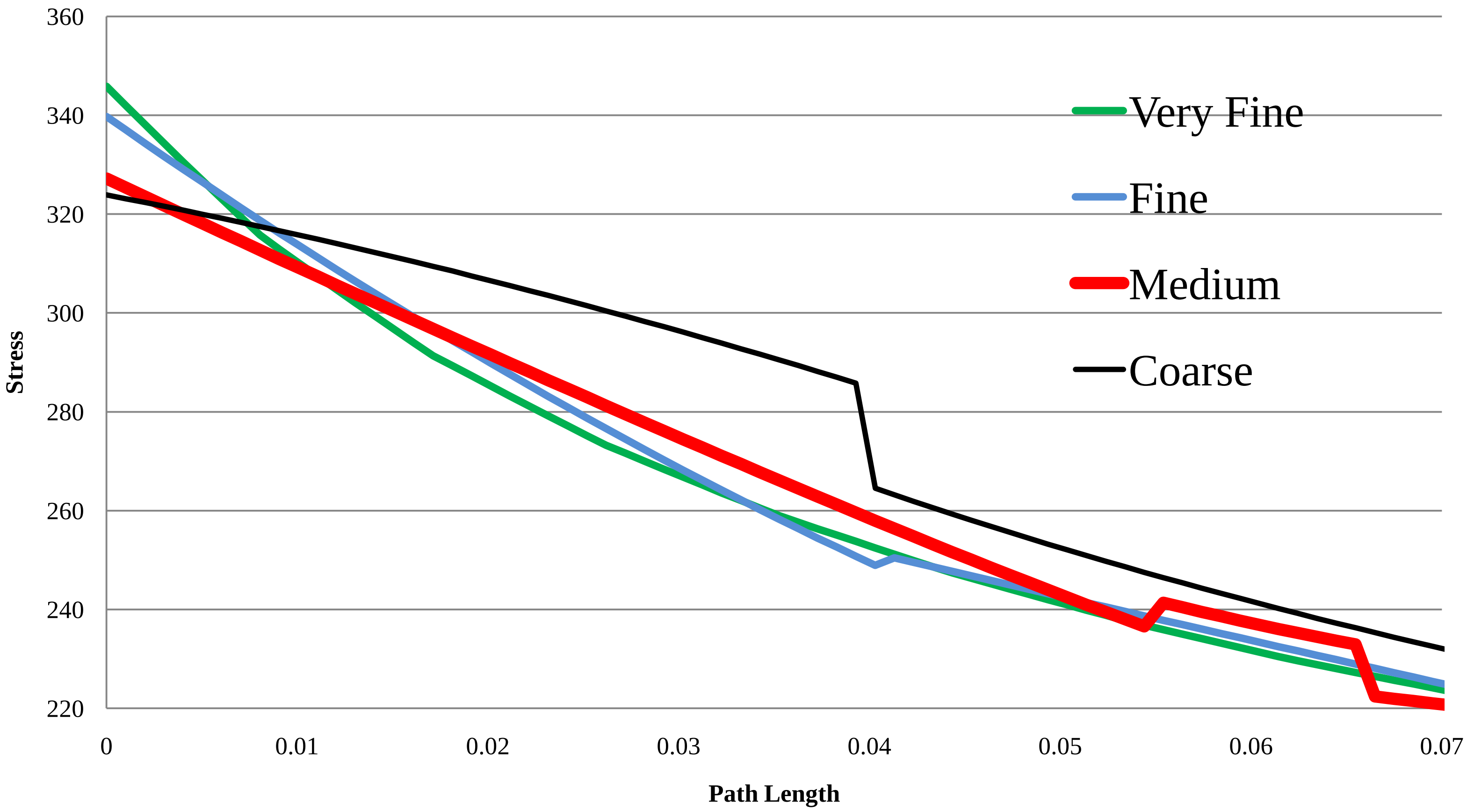
Case Study B

- Stress along path: zoom



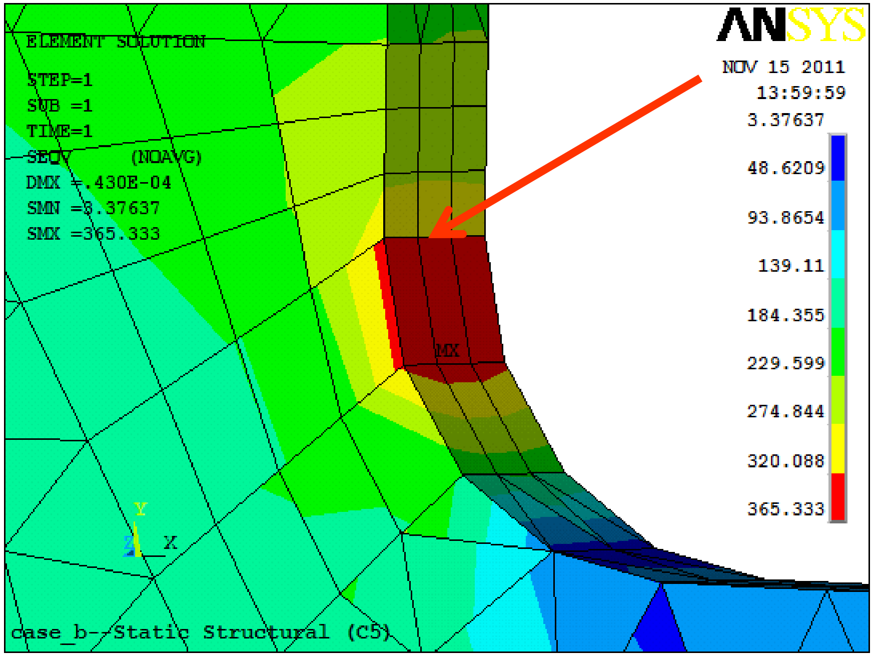
Case Study B

- Stress along path: Unaveraged Results



Case Study B

- Case Study B Conclusion:
  - Discontinuity of stress element-to-element relates to degree of mesh discretization error

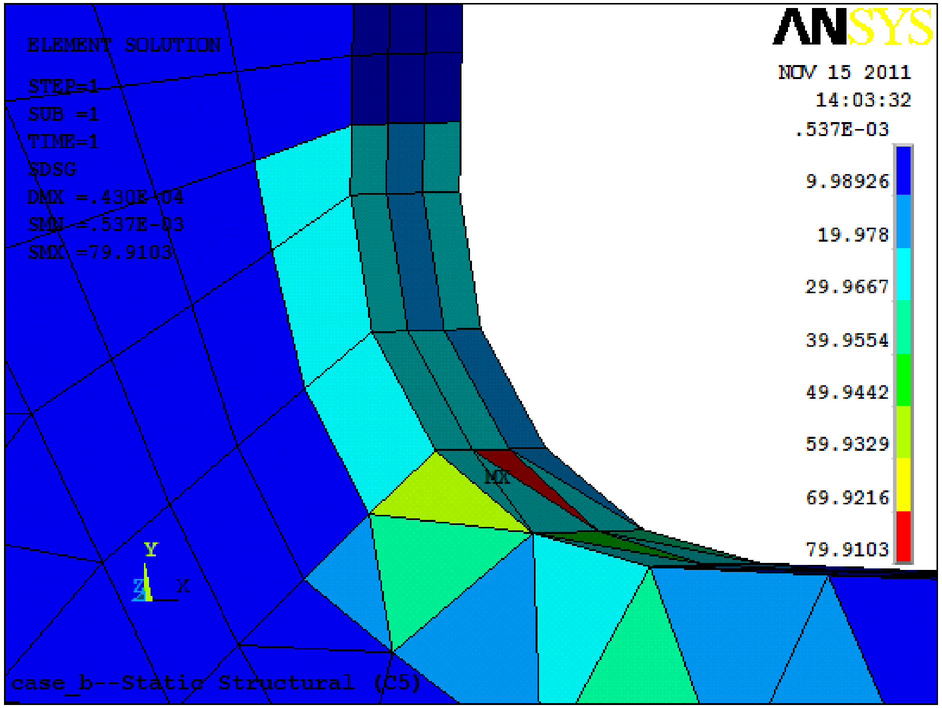


```
ELEMENT SOLUTION

STEP=1
SUB =1
TIME=1
SEQV        (NOAVG)
DMX =.430E-04
SMN =3.37637
SMX =365.333
```

Error Assessment

- Discontinuity at element boundaries is key

$\{\Delta\sigma_n^i\} = \{\sigma_n^a\} - \{\sigma_n^i\}$ Difference at boundary

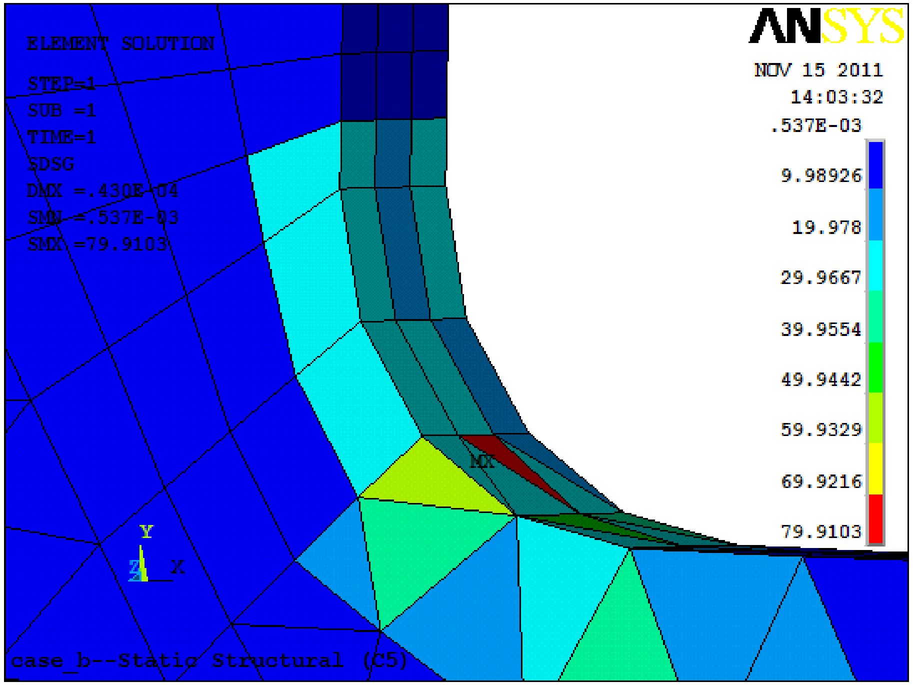


Error Assessment

- Discontinuity at element boundaries is key

$$e_i = \frac{1}{2}\int_{vol}\{\Delta\sigma\}^T[D]^{-1}\{\Delta\sigma\}d(vol)$$

- Energy difference per element
- Considers volume/stiffness



Error Assessment

- Discontinuity at element boundaries is key

$$e = \sum_{i=1}^{N} e_i$$

- Sum it over the model (selected region)

$$E = 100\left(\frac{e}{U+e}\right)^{\frac{1}{2}}$$

- Normalize it to the whole model energy
  (includes load magnitude)

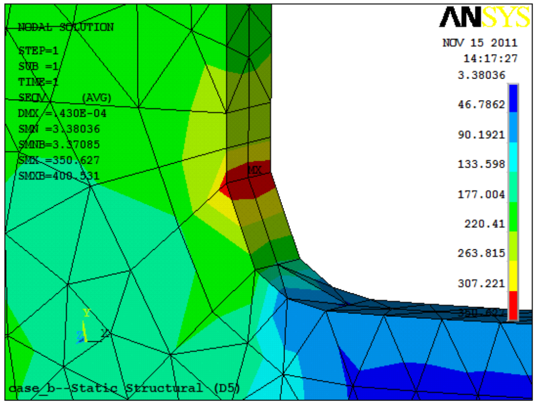
Yields a single number!

(PRERR, or Percentage error in the energy norm)

Error Assessment

- Percentage error in the energy norm (PRERR)

Coarse

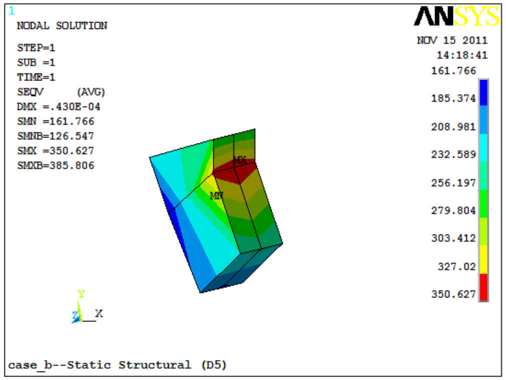


```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV      (AVG)
DMX =.430E-04
SMN =3.38036
SMNB=3.37085
SMX =350.627
SMXB=400.531
```

```
NOV 15 2011
   14:17:27
```

case_b--Static Structural (D5)

1.59



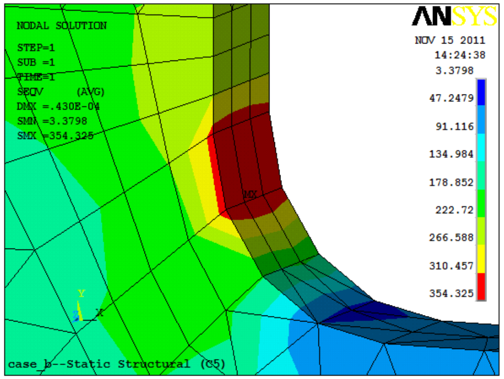
```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV      (AVG)
DMX =.430E-04
SMN =161.766
SMNB=126.547
SMX =350.627
SMXB=385.806
```

```
NOV 15 2011
   14:18:41
```

case_b--Static Structural (D5)

8.95

Medium

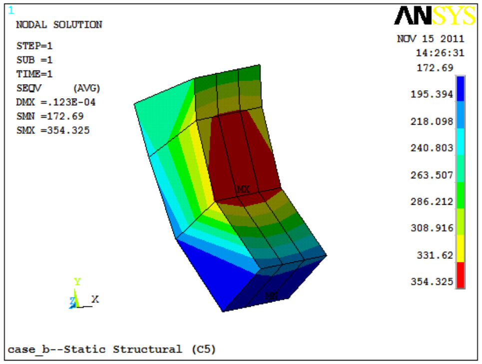


```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV      (AVG)
DMX =.430E-04
SMN =3.3798
SMX =354.325
```

```
NOV 15 2011
   14:24:38
```

case_b--Static Structural (C5)

0.797



```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV      (AVG)
DMX =.123E-04
SMN =172.69
SMX =354.325
```

```
NOV 15 2011
   14:26:31
```

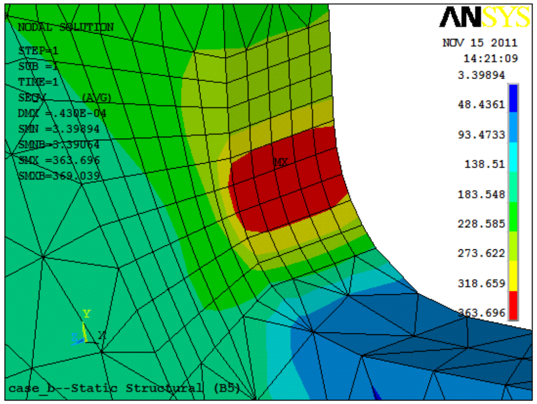
case_b--Static Structural (C5)

4.0

Error Assessment

- Percentage error in the energy norm (PRERR)

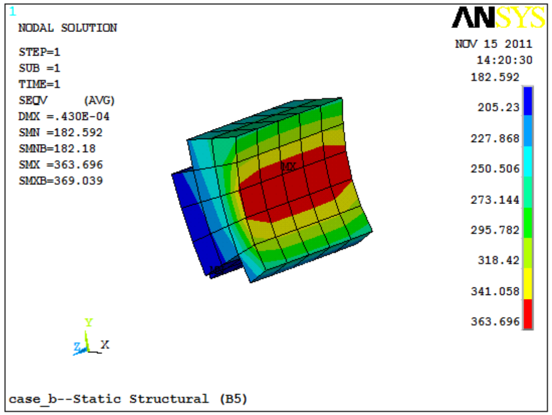
Fine



```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV       (AVG)
DMX =.430E-04
SMN =3.39894
SMNB=3.39064
SMX =363.696
SMXB=369.039
```

```
NOV 15 2011
   14:21:09
```

0.56

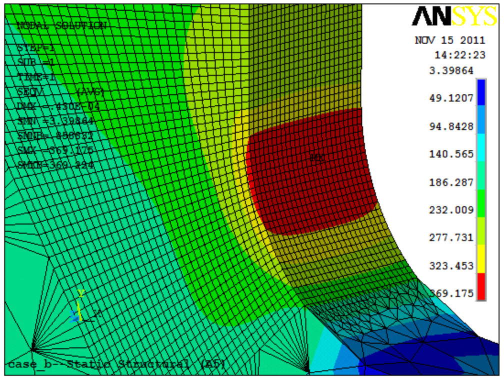


```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV       (AVG)
DMX =.430E-04
SMN =182.592
SMNB=182.18
SMX =363.696
SMXB=369.039
```

```
NOV 15 2011
   14:20:30
```

1.13

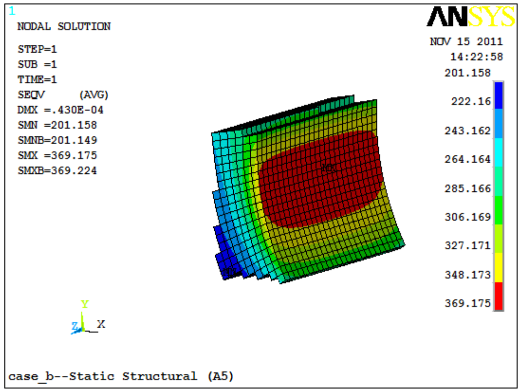
Very Fine



```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV       (AVG)
DMX =.430E-04
SMN =3.39864
SMNB=.456632
SMX =369.175
SMXB=369.224
```

```
NOV 15 2011
   14:22:23
```

0.32



```
NODAL SOLUTION

STEP=1
SUB =1
TIME=1
SEQV       (AVG)
DMX =.430E-04
SMN =201.158
SMNB=201.149
SMX =369.175
SMXB=369.224
```

```
NOV 15 2011
   14:22:58
```

0.06

Error Assessment

- SMXB
  - Checks all nodes (doesn't necessarily correspond to the MX location!)
  - Only mentioned once in Help Manual!
  - Training Classes refer to it as a "confidence band"...

$$\sigma_j^{mxb} = \max(\sigma_{j,n}^{a} + \Delta\sigma_n)$$

Root Mean Square of:
(avg. value – element value)
for each element sharing node

Average stress from
contributing elements
(what's plotted)

/EOF